Darcy friction factor formulae

From Wikipedia, the free encyclopedia

In fluid dynamics, the **Darcy friction factor formulae** are equations – based on experimental data and theory – for the Darcy friction factor. The Darcy friction factor is a dimensionless quantity used in the Darcy–Weisbach equation, for the description of friction losses in pipe flow as well as open channel flow. It is also known as the Darcy–Weisbach friction factor, resistance coefficient or simply friction factor and is four times larger than the Fanning friction factor.[1]

Contents

Flow regime

Which friction factor formula may be applicable depends upon the type of flow that exists:

- Laminar flow
- Transition between laminar and turbulent flow
- Fully turbulent flow in smooth conduits
- Fully turbulent flow in rough conduits
- Free surface flow.

Laminar flow

The Darcy friction factor for laminar flow in a circular pipe (Reynolds number less than 2320) is given by the following formula:

$$f = \frac{64}{\mathrm{Re}}$$

where:

- f is the Darcy friction factor

- Re is the Reynolds number.

Transition flow

Transition (neither fully laminar nor fully turbulent) flow occurs in the range of Reynolds numbers between 2300 and 4000. The value of the Darcy friction factor is subject to large uncertainties in this flow regime.

Turbulent flow in smooth conduits

The Blasius correlation is the simplest equation for computing the Darcy friction factor. Because the Blasius correlation has no term for pipe roughness, it is valid only to smooth pipes. However, the Blasius correlation is sometimes used in rough pipes because of its simplicity. The Blasius correlation is valid up to the Reynolds number 100000.

Turbulent flow in rough conduits

The Darcy friction factor for fully turbulent flow (Reynolds number greater than 4000) in rough conduits is given by the Colebrook equation.

Free surface flow

The last formula in the *Colebrook equation* section of this article is for free surface flow. The approximations elsewhere in this article are not applicable for this type of flow.

Choosing a formula

Before choosing a formula it is worth knowing that in the paper on the Moody chart, Moody stated the accuracy is about ±5% for smooth pipes and ±10% for rough pipes. If more than one formula is applicable in the flow regime under consideration, the choice of formula may be influenced by one or more of the following:

- Required precision
- Speed of computation required
- Available computational technology:
  - calculator (minimize keystrokes)
  - spreadsheet (single-cell formula)
  - programming/scripting language (subroutine).

Colebrook–White equation

The phenomenological Colebrook–White equation (or Colebrook equation) expresses the Darcy friction factor *f* as a function of Reynolds number Re and pipe relative roughness ε / D_h, fitting the data of experimental studies of turbulent flow in smooth and rough pipes.[2][3] The equation can be used to (iteratively) solve for the Darcy–Weisbach friction factor *f*.

For a conduit flowing completely full of fluid at Reynolds numbers greater than 4000, it is expressed as:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D_\mathrm{h}} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

or

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{14.8R_\mathrm{h}} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

where:

- f is the Darcy friction factor
- Roughness height, ε (m, ft)
- Hydraulic diameter, D_h (m, ft) – For fluid-filled, circular conduits, D_h = D = inside diameter
- Hydraulic radius, R_h (m, ft) – For fluid-filled, circular conduits, R_h = D/4 = (inside diameter)/4
- Re is the Reynolds number

Note: Some sources use a constant of 3.71 in the denominator for the roughness term in the first equation above.[4]

Solving

The Colebrook equation is usually solved numerically due to its implicit nature. Recently, the Lambert W function has been employed to obtain explicit reformulation of the Colebrook equation.[5]

You can solve the Colebrook equation by iteration using the Newton–Raphson method. An example is provided in C# here.[6]

Expanded forms

Additional, mathematically equivalent forms of the Colebrook equation are:

$$\frac{1}{\sqrt{f}} = 1.7384\ldots - 2\log_{10}\left(\frac{2\varepsilon}{D_\mathrm{h}} + \frac{18.574}{\mathrm{Re}\sqrt{f}}\right)$$

where:

1.7384... = 2 log (2 × 3.7) = 2 log (7.4)
18.574 = 2.51 × 3.7 × 2

and

$$\frac{1}{\sqrt{f}} = 1.1364\ldots + 2\log_{10}(D_\mathrm{h}/\varepsilon) - 2\log_{10}\left(1 + \frac{9.287}{\mathrm{Re}(\varepsilon/D_\mathrm{h})\sqrt{f}}\right)$$

or

$$\frac{1}{\sqrt{f}} = 1.1364\ldots - 2\log_{10}\left(\frac{\varepsilon}{D_\mathrm{h}} + \frac{9.287}{\mathrm{Re}\sqrt{f}}\right)$$

where:

1.1364... = 1.7384... − 2 log (2) = 2 log (7.4) − 2 log (2) = 2 log (3.7)
9.287 = 18.574 / 2 = 2.51 × 3.7.

The additional equivalent forms above assume that the constants 3.7 and 2.51 in the formula at the top of this section are exact. The constants are probably values which were rounded by Colebrook during his curve fitting; but they are effectively treated as exact when comparing (to several decimal places) results from explicit formulae (such as those found elsewhere in this article) to the friction factor computed via Colebrook's implicit equation.

Equations similar to the additional forms above (with the constants rounded to fewer decimal places, or perhaps shifted slightly to minimize overall rounding errors) may be found in various references. It may be helpful to note that they are essentially the same equation.

Free surface flow

Another form of the Colebrook-White equation exists for free surfaces. Such a condition may exist in a pipe that is flowing partially full of fluid. For free surface flow:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{12R_\mathrm{h}} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right).$$

Approximations of the Colebrook equation

Haaland equation

The *Haaland equation* was proposed by Norwegian Institute of Technology professor Haaland in 1984. It is used to solve directly for the Darcy–Weisbach friction factor *f* for a full-flowing circular pipe. It is an approximation of the implicit Colebrook–White equation, but the discrepancy from experimental data is well within the accuracy of the data. It was developed by S. E. Haaland in 1983.

The Haaland equation is defined as:

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$$ [7]

where:

- f is the Darcy friction factor
- ε/D is the relative roughness
- Re is the Reynolds number.

Swamee–Jain equation

The Swamee–Jain equation is used to solve directly for the Darcy–Weisbach friction factor *f* for a full-flowing circular pipe. It is an approximation of the implicit Colebrook–White equation.

$$f = 0.25\left[\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^{-2}$$

[8]

where *f* is a function of:

- Roughness height, ε (m, ft)
- Pipe diameter, *D* (m, ft)
- Reynolds number, *Re* (unitless).

Serghides's solution

Serghides's solution is used to solve directly for the Darcy–Weisbach friction factor *f* for a full-flowing circular pipe. It is an approximation of the implicit Colebrook–White equation. It was derived using Steffensen's method.[9]

The solution involves calculating three intermediate values and then substituting those values into a final equation.

$$A = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{12}{\mathrm{Re}}\right)$$

$$B = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51A}{\mathrm{Re}}\right)$$

$$C = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51B}{\mathrm{Re}}\right)$$

$$\frac{1}{\sqrt{f}} = \left(A - \frac{(B-A)^2}{C-2B+A}\right)$$

where *f* is a function of:

- Roughness height, ε (m, ft)
- Pipe diameter, *D* (m, ft)
- Reynolds number, Re (unitless).

The equation was found to match the Colebrook–White equation within 0.0023% for a test set with a 70-point matrix consisting of ten relative roughness values (in the range 0.00004 to 0.05) by seven Reynolds numbers (2500 to 10^8).

Goudar–Sonnad equation

Goudar equation is the most accurate approximation to solve directly for the Darcy–Weisbach friction factor *f* for a full-flowing circular pipe. It is an approximation of the implicit Colebrook–White equation. Equation has the following form[10]

$$a = \frac{2}{\ln(10)}$$
$$b = \frac{\varepsilon/D}{3.7}$$
$$d = \frac{\ln(10)Re}{5.02}$$
$$s = bd + \ln(d)$$
$$q = s^{s/(s+1)}$$
$$g = bd + \ln\frac{d}{q}$$
$$z = \ln\frac{q}{g}$$
$$D_{LA} = z\frac{g}{g+1}$$
$$D_{CFA} = D_{LA}\left(1 + \frac{z/2}{(g+1)^2 + (z/3)(2g-1)}\right)$$
$$\frac{1}{\sqrt{f}} = a\left[\ln\left(\frac{d}{q}\right) + D_{CFA}\right]$$

where *f* is a function of:

- Roughness height, ε (m, ft)
- Pipe diameter, *D* (m, ft)
- Reynolds number, Re (unitless).

Brkić solution

Brkić shows one approximation of the Colebrook equation based on the Lambert W-function[11]

$$S = ln\frac{Re}{1.816\ln\frac{1.1\mathrm{Re}}{\ln(1+1.1\mathrm{Re})}}$$

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.71} + \frac{2.18S}{\mathrm{Re}}\right)$$

where Darcy friction factor *f* is a function of:

- Roughness height, ε (m, ft)
- Pipe diameter, *D* (m, ft)
- Reynolds number, Re (unitless).

The equation was found to match the Colebrook–White equation within 3.15%.

Blasius correlations

Early approximations by Paul Richard Heinrich Blasius in terms of the Moody friction factor are given in one article of 1913:[12]

$$f = .316\mathrm{Re}^{-\frac{1}{4}}.$$

Johann Nikuradse in 1932 proposed that this corresponds to a power law correlation for the fluid velocity profile.

Mishra and Gupta in 1979 proposed a correction for curved or helically coiled tubes, taking into account the equivalent curve radius, R_c:[13]

$$f = 0.316\mathrm{Re}^{-\frac{1}{4}} + 0.0075\sqrt{\frac{D}{2R_c}},$$

with,

$$R_c = R\left[1 + \left(\frac{H}{2\pi R}\right)^2\right]$$

where *f* is a function of:

- Pipe diameter, *D* (m, ft)
- Curve radius, *R* (m, ft)
- Helicoidal pitch, *H* (m, ft)
- Reynolds number, *Re* (unitless)

valid for:

- $Re_{tr} < Re < 10^5$
- $6.7 < 2R_c/D < 346.0$
- $0 < H/D < 25.4$

Table of Approximations

The following table lists historical approximations where:[14]

- Re, Reynolds number (unitless);
- λ, Darcy friction factor (dimensionless);
- ε, roughness of the inner surface of the pipe (dimension of length);
- *D*, inner pipe diameter;
- $\log(x)$ is the base-10 logarithm.

Note that the Churchill equation [15] (1977) is the only one that returns a correct value for friction factor in the laminar flow region (Reynolds number < 2300). All of the others are for transitional and turbulent flow only.

Table of Colebrook equation approximations

Equation	Author	Year	Ref
$\lambda = .0055(1 + (2 \times 10^4 \cdot \frac{\varepsilon}{D} + \frac{10^6}{Re})^{\frac{1}{3}})$	Moody	1947	
$\lambda = .094(\frac{\varepsilon}{D})^{0.225} + 0.53(\frac{\varepsilon}{D}) + 88(\frac{\varepsilon}{D})^{0.44} \cdot Re^{-\Psi}$ where $\Psi = 1.62(\frac{\varepsilon}{D})^{0.134}$	Wood	1966	
$\frac{1}{\sqrt{\lambda}} = -2\log(\frac{\varepsilon}{3.715D} + \frac{15}{Re})$	Eck	1973	
$\frac{1}{\sqrt{\lambda}} = -2\log(\frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}})$	Jain and Swamee	1976	
$\frac{1}{\sqrt{\lambda}} = -2\log((\frac{\varepsilon}{3.71D}) + (\frac{7}{Re})^{0.9})$	Churchill	1973	
$\frac{1}{\sqrt{\lambda}} = -2\log((\frac{\varepsilon}{3.715D}) + (\frac{6.943}{Re})^{0.9}))$	Jain	1976	
$\lambda = 8[(\frac{8}{Re})^{12} + \frac{1}{(\Theta_1 + \Theta_2)^{1.5}})]^{\frac{1}{12}}$ where $\Theta_1 = [-2.457\ln[(\frac{7}{Re})^{0.9} + 0.27\frac{\varepsilon}{D}]]^{16}$ $\Theta_2 = (\frac{37530}{Re})^{16}$	Churchill	1977	
$\frac{1}{\sqrt{\lambda}} = -2\log[\frac{\varepsilon}{3.7065D} - \frac{5.0452}{Re}\log(\frac{1}{2.8257}(\frac{\varepsilon}{D})^{1.1098} + \frac{5.8506}{Re^{0.8981}})]$	Chen	1979	
$\frac{1}{\sqrt{\lambda}} = 1.8\log[\frac{Re}{0.135Re(\frac{\varepsilon}{D}) + 6.5}]$	Round	1980	
$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{5.158log(\frac{Re}{7})}{Re\left(1 + \frac{Re^{0.52}}{29}(\frac{\varepsilon}{D})^{0.7}\right)}\right)$	Barr	1981	
$\frac{1}{\sqrt{\lambda}} = -2\log[\frac{\varepsilon}{3.7D} - \frac{5.02}{Re}\log(\frac{\varepsilon}{3.7D} - \frac{5.02}{Re}\log(\frac{\varepsilon}{3.7D} + \frac{13}{Re}))]$ or $\frac{1}{\sqrt{\lambda}} = -2\log[\frac{\varepsilon}{3.7D} - \frac{5.02}{Re}\log(\frac{\varepsilon}{3.7D} + \frac{13}{Re})]$	Zigrang and Sylvester	1982	
$\frac{1}{\sqrt{\lambda}} = -1.8\log\left[\left(\frac{\varepsilon}{3.7D}\right)^{1.11} + \frac{6.9}{Re}\right]$	Haaland[7]	1983	
$\lambda = [\Psi_1 - \frac{(\Psi_2 - \Psi_1)^2}{\Psi_3 - 2\Psi_2 + \Psi_1}]^{-2}$ or			

$\lambda = [4.781 - \frac{(\Psi_1 - 4.781)^2}{\Psi_2 - 2\Psi_1 + 4.781}]^{-2}$ where $\Psi_1 = -2\log(\frac{\varepsilon}{3.7D} + \frac{12}{Re})$ $\Psi_2 = -2\log(\frac{\varepsilon}{3.7D} + \frac{2.51\Psi_1}{Re})$ $\Psi_3 = -2\log(\frac{\varepsilon}{3.7D} + \frac{2.51\Psi_2}{Re})$	Serghides	1984
$\frac{1}{\sqrt{\lambda}} = -2\log(\frac{\varepsilon}{3.7D} + \frac{95}{Re^{0.983}} - \frac{96.82}{Re})$	Manadilli	1997
$\frac{1}{\sqrt{\lambda}} = -2\log\{\frac{\varepsilon}{3.7065D} - \frac{5.0272}{Re}\log[\frac{\varepsilon}{3.827D} - \frac{4.657}{Re}\log((\frac{\varepsilon}{7.7918D})^{0.9924} + (\frac{5.3326}{208.815 + Re})^{0.9345})]\}$	Monzon, Romeo, Royo	2002
$\frac{1}{\sqrt{\lambda}} = 0.8686\ln[\frac{0.4587Re}{(S-0.31)^{\frac{S}{(S+1)}}}]$ where: $S = 0.124Re\frac{\varepsilon}{D} + \ln(0.4587Re)$	Goudar, Sonnad	2006
$\frac{1}{\sqrt{\lambda}} = 0.8686\ln[\frac{0.4587Re}{(S-0.31)^{\frac{S}{(S+0.9633)}}}]$ where: $S = 0.124Re\frac{\varepsilon}{D} + \ln(0.4587Re)$	Vatankhah, Kouchakzadeh	2008
$\frac{1}{\sqrt{\lambda}} = \alpha - [\frac{\alpha + 2\log(\frac{B}{Re})}{1 + \frac{2.18}{B}}]$ where $\alpha = \frac{(0.744\ln(Re)) - 1.41}{(1 + 1.32\sqrt{\frac{\varepsilon}{D}})}$ $B = \frac{\varepsilon}{3.7D}Re + 2.51\alpha$	Buzzelli	2008
$\lambda = \frac{6.4}{(\ln(Re) - \ln(1 + .01Re\frac{\varepsilon}{D}(1 + 10\sqrt{\frac{\varepsilon}{D}})))^{2.4}}$	Avci, Kargoz	2009
$\lambda = \frac{0.2479 - 0.0000947(7 - \log Re)^4}{(\log(\frac{\varepsilon}{3.615D} + \frac{7.366}{Re^{0.9142}}))^2}$	Evangleids, Papaevangelou, Tzimopoulos	2010

References

1. Manning, Francis S.; Thompson, Richard E. (1991). *Oilfield Processing of Petroleum. Vol. 1: Natural Gas*. PennWell Books. ISBN 0-87814-343-2., 420 pages. See page 293.
2. Colebrook, C. F. and White, C. M. (1937). "Experiments with Fluid Friction in Roughened Pipes". *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences* **161** (906): 367–381. Bibcode:1937RSPSA.161..367C. doi:10.1098/rspa.1937.0150. Often erroneously cited as the source of the Colebrook-White equation. This is partly because Colebrook (in a footnote in his 1939 paper) acknowledges his debt to White for suggesting the mathematical method by which the smooth and rough pipe correlations could be combined.
3. Colebrook, C. F. (February 1939). "Turbulent flow in pipes, with particular reference to the transition region between smooth and rough pipe laws". *Journal of the Institution of Civil Engineers* (London).
4. VDI Heat Atlas second edition page 1058 (ISBN 978-3-540-77876-9)
5. More, A. A. (2006). "Analytical solutions for the Colebrook and White equation and for pressure drop in ideal gas flow in pipes". *Chemical Engineering Science* **61** (16): 5515–5519. doi:10.1016/j.ces.2006.04.003.
6. Métodos Numéricos con C# (http://processimulation.blogspot.no/2014/06/metodos-numericos-con-c-en-esta.html)
7. BS Massey Mechanics of Fluids 7th Ed ISBN 0-412-34280-4
8. P.K. Swami, A.K. Jaine, Explicit equations for pipeflow problems, J Hydraulics Div, Proc ASCE (1976), pp. 657–664 May
9. Serghides, T.K (1984). "Estimate friction factor accurately". *Chemical Engineering Journal* **91**(5): 63–64.
10. Goudar, C.T., Sonnad, J.R. (August 2008). "Comparison of the iterative approximations of the Colebrook–White equation". *Hydrocarbon Processing* **Fluid Flow and**

Rotating Equipment Special Report(August 2008): 79–83.
11. Brkić, Dejan (2011). "An Explicit Approximation of Colebrook's equation for fluid flow friction factor". *Petroleum Science and Technology* **29** (15): 1596–1602. doi:10.1080/10916461003620453.
12. Trinh, On the Blasius correlation for friction factors, p. 1 (http://arxiv.org/ftp/arxiv/papers/1007/1007.2466.pdf)
13. Adrian Bejan, Allan D. Kraus, Heat transfer handbook, John Wiley & Sons, 2003
14. Beograd, Dejan Brkić (March 2012). "Determining Friction Factors in Turbulent Pipe Flow". *Chemical Engineering*: 34–39.(subscription required)
15. Churchill, S.W. (November 7, 1977). "Friction-factor equation spans all fluid-flow regimes". *Chemical Engineering*: 91–92.

Further reading

- Colebrook, C.F. (February 1939). "Turbulent flow in pipes, with particular reference to the transition region between smooth and rough pipe laws". *Journal of the Institution of Civil Engineers* (London). doi:10.1680/ijoti.1939.13150.
  For the section which includes the free-surface form of the equation – "Computer Applications in Hydraulic Engineering" (5th ed.). Haestad Press. 2002., p. 16.
- Haaland, SE (1983). "Simple and Explicit Formulas for the Friction Factor in Turbulent Flow". *Journal of Fluids Engineering* (ASME) **105** (1): 89–90. doi:10.1115/1.3240948.
- Swamee, P.K.; Jain, A.K. (1976). "Explicit equations for pipe-flow problems". *Journal of the Hydraulics Division* (ASCE) **102** (5): 657–664.
- Serghides, T.K (1984). "Estimate friction factor accurately". *Chemical Engineering* **91** (5): 63–64. – Serghides' solution is also mentioned here (http://www.cheresources.com/colebrook2.shtml).
- Moody, L.F. (1944). "Friction Factors for Pipe Flow". *Transactions of the ASME* **66** (8): 671–684.
- Brkić, Dejan (2011). "Review of explicit approximations to the Colebrook relation for flow friction". *Journal of Petroleum Science and Engineering* **77** (1): 34–48. doi:10.1016/j.petrol.2011.02.006.
- Brkić, Dejan (2011). "W solutions of the CW equation for flow friction". *Applied Mathematics Letters* **24** (8): 1379–1383. doi:10.1016/j.aml.2011.03.014.

External links

- Web-based calculator of Darcy friction factors by Serghides' solution. (http://www.calctool.org/CALC/eng/civil/friction_factor)
- Open source pipe friction calculator. (http://pfcalc.sourceforge.net)